Paper Reference(s)

6680

Edexcel GCE

Mechanics M4

Advanced Level

Specimen Paper

Time: 1 hour 30 minutes

<u>**Materials required for examination**</u>   <u>**Items included with question papers**</u>
Answer Book (AB16)   Nil
Mathematical Formulae (Lilac)
Graph Paper (ASG2)

Candidates may use any calculator EXCEPT those with the facility for symbolic algebra, differentiation and/or integration. Thus candidates may NOT use calculators such as the Texas Instruments TI 89, TI 92, Casio CFX 9970G, Hewlett Packard HP 48G.

Instructions to Candidates

In the boxes on the answer book, write the name of the examining body (Edexcel), your centre number, candidate number, the unit title (Mechanics M4), the paper reference (6680), your surname, other name and signature.
Whenever a numerical value of g is required, take $g = 9.8 \text{ m s}^{-2}$.
When a calculator is used, the answer should be given to an appropriate degree of accuracy.

Information for Candidates

A booklet 'Mathematical Formulae and Statistical Tables' is provided.
Full marks may be obtained for answers to ALL questions.
This paper has seven questions.

Advice to Candidates

You must ensure that your answers to parts of questions are clearly labelled.
You must show sufficient working to make your methods clear to the Examiner. Answers without working may gain no credit.

1. A particle *P* of mass 2 kg moves in a straight line along a smooth horizontal plane. The only horizontal force acting on *P* is a resistance of magnitude $4v$ N, where v m s^{-1} is its speed. At time $t = 0$ s, *P* has a speed of 5 m s^{-1}. Find v in terms of t.

    (6)

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2. **Figure 1**
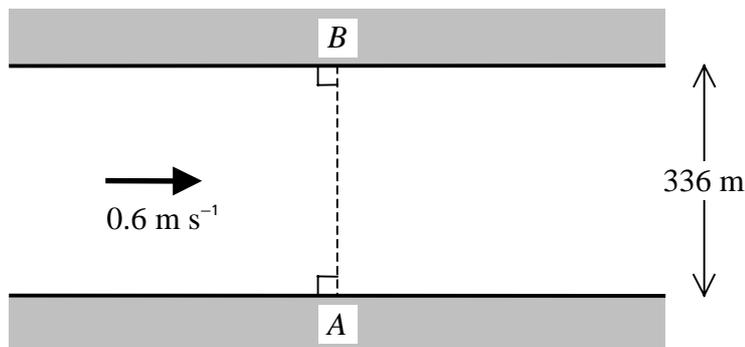
    A girl swims in still water at 1 m s^{-1}. She swims across a river which is 336 m wide and is flowing at 0.6 m s^{-1}. She sets off from a point *A* on one bank and lands at a point *B*, which is directly opposite *A*, on the other bank as shown in Fig. 1. Find

    (*a*) the direction, relative to the earth, in which she swims,

    (3)

    (*b*) the time that she takes to cross the river.

    (3)

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3. A ball of mass *m* is thrown vertically upwards from the ground. When its speed is *v* the magnitude of the air resistance is modelled as being mkv^2, where *k* is a positive constant. The ball is projected with speed $\sqrt{\dfrac{g}{k}}$.

    By modelling the ball as a particle,

    (*a*) find the greatest height reached by the ball.

    (9)

    (*b*) State one physical factor which is ignored in this model.

    (1)

4.

Figure 2
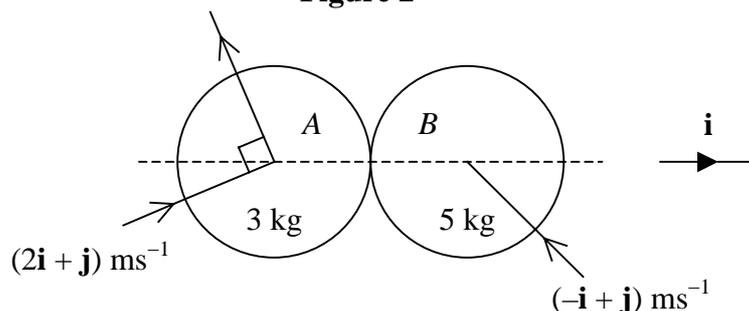
Two smooth uniform spheres *A* and *B*, of equal radius, are moving on a smooth horizontal plane. Sphere *A* has mass 3 kg and velocity $(2\mathbf{i} + \mathbf{j})$ m s^{-1}, and sphere *B* has mass 5 kg and velocity $(-\mathbf{i} + \mathbf{j})$ m s^{-1}. When the spheres collide the line joining their centres is parallel to **i**, as shown in Fig. 2.

Given that the direction of *A* is deflected through a right angle by the collision, find

(*a*) the velocity of *A* after the collision,

(5)

(*b*) the coefficient of restitution between the spheres.

(6)

5. An elastic string spring of modulus 2*mg* and natural length *l* is fixed at one end. To the other end is attached a mass *m* which is allowed to hang in equilibrium. The mass is then pulled vertically downwards through a distance *l* and released from rest. The air resistance is modelled as having magnitude 2*mωv*, where *v* is the speed of the particle and $\omega = \sqrt{\dfrac{g}{l}}$. The particle is at distance *x* from its equilibrium position at time *t*.

(*a*) Show that $\dfrac{\mathrm{d}^2 x}{\mathrm{d}t^2} + 2\omega \dfrac{\mathrm{d}x}{\mathrm{d}t} + 2\omega^2 x = 0$.

(7)

(*b*) Find the general solution of this differential equation.

(4)

(*c*) Hence find the period of the damped harmonic motion.

(1)

Turn Over

6. Two horizontal roads cross at right angles. One is directed from south to north, and the other from east to west. A tractor travels north on the first road at a constant speed of 6 m s^{-1} and at noon is 200 m south of the junction. A car heads west on the second road at a constant speed of 24 m s^{-1} and at noon is 960 m east of the junction.

(*a*) Find the magnitude and direction of the velocity of the car relative to the tractor.

(6)

(*b*) Find the shortest distance between the car and the tractor.

(8)

7. **Figure 3**
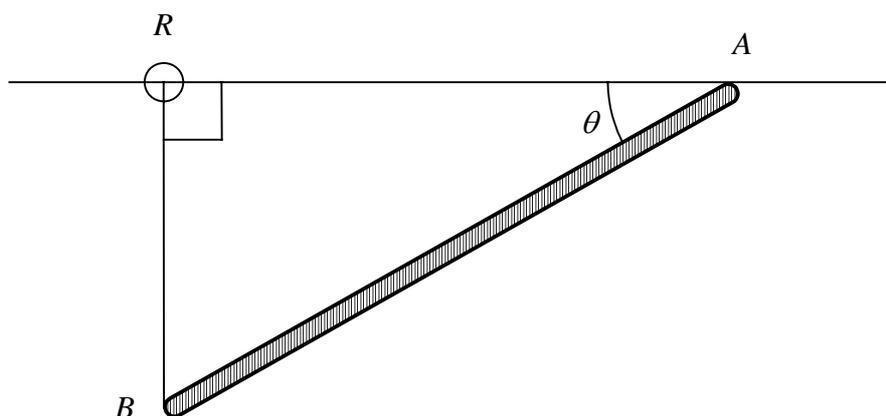
A uniform rod *AB* has mass *m* and length 2*a*. The end *A* is smoothly hinged at a fixed point on a fixed straight horizontal wire. A smooth light ring *R* is threaded on the wire. The ring *R* is attached by a light elastic string, of natural length *a* and modulus of elasticity *mg*, to the end *B* of the rod. The end *B* is always vertically below *R* and angle $\angle RAB = \theta$, as shown in Fig. 3.

(*a*) Show that the potential energy of the system is

$$mga\left(2\sin^2\theta - 3\sin\theta\right) + \text{constant}.$$

(6)

(*b*) Hence determine the value of θ, $\theta < \dfrac{\pi}{2}$, for which the system is in equilibrium.

(5)

(c) Determine whether this position of equilibrium is stable or unstable.

(5)

END